Probabilistic quantum error correction codes for general noise channels

Doctoral dissertation summary

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Summary

The main aim of this dissertation was to develop a theory of probabilistic quantum error correction codes for general noise channels. By general noise channel, we define quantum operations with no particular structure that can be described by open quantum systems and the Stinespring dilation theorem. The conducted analysis showed that probabilistic codes are suitable for very noisy quantum systems. The results of this thesis confirm the formulated **Hypothesis**:

The usage of probabilistic quantum error correction codes can improve the quality of quantum systems disturbed by general noise channels.

In more detail, we generalized the Kill-Laflamme theorem. We established the necessary and sufficient conditions to check if a given noise channel is probabilistically correctable. We used these conditions to show that the probabilistic codes, in comparison to the deterministic codes, are able to correct noise channels from a larger set. A clear separation between probabilistic and deterministic codes was observed for: Schur noise channels and for random quantum channels. Each of these examples indicated a trade-off between the probability of successful error correction and the quality of the code. We proved that noise channels with "low" Choi rank are probabilistically correctable.

As an engineering aspect confirming the dissertation hypothesis, we created a numerically efficient algorithm to construct an approximate probabilistic codes. This construction has two essential properties. First, it almost surely returns a perfect probabilistic error-correcting code for any quantum channel with "low" Choi rank. Second, for any other quantum channel, this construction provides a scheme with a relatively high value of the fidelity function. We proved the first claim, while the second claim we checked numerically by using randomly sampled quantum noise channels.

The numerical investigation was possible due to advancement in the methods of generating random quantum channels. In this dissertation, we showed the equivalence of sampling techniques based on different representations of quantum channels. In particular, the method based on the Kraus representation happened to be the most suitable for numerical simulation, providing appropriate diversity of sampled channels and having a simple implementation.

The work consists of six chapters. The first Chapter introduces the theory of quantum error correction and the motivation for my research. The second Chapter presents the necessary mathematical framework. The rest of the dissertation is based on two published articles and my self-directed unpublished results. The third Chapter concerns the overview of random quantum operations. First, we explore the methods of generating random channels and then analyze their properties. Next, the forth Chapter focuses on the pQEC codes. Here, we present a general problem

formulation, motivation and necessary theoretical framework. Most importantly, we show the advantages of using the pQEC codes. In the fifth Chapter, basing on the pQEC procedure, we create an efficient method of constructing approximate probabilistic quantum error correction codes. To show the potential of this approach, we test the proposed procedure on randomly generated quantum channels. This chapter is my contribution to the dissertation. In the last Chapter we conclude this dissertation.

Published work

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9. Lewandowska, P., Krawiec, A., Kukulski, R., Pawela, Ł., & Puchała, Z. (2021). *On the optimal certification of von Neumann measurements.* Scientific reports, 11(1), 1-16., arXiv:2009.06776, DOI:10.1038/s41598-021-81325-1,
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Preprints

1. Kukulski, R., & Glos, A. (2020). *Comment to Spatial Search by Quantum Walk is Optimal for Almost all Graphs.* arXiv preprint arXiv:2009.13309.
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Publications related to the dissertation are highlighted in **bold font**.