Report on *"Theory and Applications of Hybrid Quantum-Classical Optimization Algorithms"*, the PhD Thesis of **mgr Ludmila BOTELHO**, supervised by **dr hab. Jaroslaw MISZCZAK, prof IITiS**

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1 Overview

The thesis of **mgr Ludmila BOTELHO** (hereafter referred to as "the author") is titled "Theory and Applications of Hybrid Quantum-Classical Optimization Algorithms". Having read (and corrected in detail) all of the thesis, firstly I would say that the title does indeed match the content (this is important!)

2 Scope and Results

Following a short introduction, the scene is set in Chapter 2 ("Preliminaries") for the three main pieces of research:

Chapter 3 "Error mitigation for QAOA using post-selection"

Chapter 4 "Hamiltonian-oriented homotopy QAOA"

Chapter 4 "Applications of quantum annealing for music theory"

The themes of the dissertation are topical, and at the cutting edge of research in the field.

2.1 Preliminaries

This Chapter is comprehensive, well written, easy to read, while explaining many ideas from mathematics, physics and computer science in an accessible way. Herein,

- The author explains relevant ideas from mathematics (graph theory), physics (qubits, Unitary operators, measurement, Ising model), computer science (computational complexity).
- The author explains some of the application areas/problems (Max-Cut, Travelling Salesperson, Job Scheduling,...).
- Relevant classical approaches to solving the problems mentioned are covered - ILP (Integer Linear Programming) and QUBO (Quadratic Unconstrained Binary Optimization).
- The main ideas in a number of quantum or quantum-classical hybrid algorithms are explained. This includes AQC (Adiabatic Quantum Computing), QA (Quantum Annealing), VQA (Variational Quantum Algorithms), and QAOA (Quantum Approximation Optimization Algorithms).

2.2 Error mitigation for QAOA using post-selection

In this chapter, new procedures are developed and applied for error mitigation in VQA. In particular, post-selection is implemented in the QAOA (Quantum Alternating Operator Ansatz), and applied to the Travelling Salesperson Problem. The different post-selection strategies presented are

- $k-$hot
- One-hot
- Domain-wall
- Binary and Gray encoding

Table 3.1 presents a comprehensive list of the results, which are too detailed to list here.

It is shown that for certain noise models (amplitude damping, depolarizing, bit-flip), an improvement is obtained using mid-circuit post-selection.

2.3 Hamiltonian-oriented homotopy QAOA

This chapter discusses the (novel) idea of solving combinatorial optimization problems using Hamiltonian Oriented Homotopy Quantum Alternating Operator Ansatz (HOHo-QAOA). The method considers the behaviour of a parameterized convex sum of "mixer" Hamiltonian and "objective" Hamiltonian. An application is shown to the Max-Cut problem. This homotopy-inspired technique is shown to give algorithmic improvements in certain cases.

2.4 Applications of quantum annealing for music theory

This is the chapter that I found the most interesting and novel, in that very few authors (if any) have applied quantum computing to music. The author shows how quantum annealing and QUBO can be used to compose music. The examples are nice, and the rules governing transitions between notes are carefully explained. Practical examples are presented where the author uses his technique to design new melodies. Further considerations are given to creation of rythm using the model.

Compared to other (technical/scientific) application areas discussed, it is refreshing to see quantum computing applied in the arts: Of course it will be more subjective - and less scientific - to try to analyze if the resulting output (in the case of music generation) is "musical" or "artistic". The author stays away from claims in this area, but it is certainly plausible (and maybe probable) that nice music can be composed using the techniques presented.

3 Importance and Novelty

In the main three chapters of the work, the scientific problems are posed clearly. Some of these problems do not have "just one" method of solution: Nonetheless, the author presents solutions that are novel, and also presents awareness that there are other approaches. The work is, throughout, and to use a phrase used by the author, "state of the art". The author displays a comprehensive knowledge of previous and current work in the area, on which the thesis builds.

4 Weaknesses

If any weakness can be pointed out, I think I would say, the thesis is a little disjointed (certainly in the applications). Chapter 5 (which is nonetheless wonderful work) seems disconnected from the previous chapters. The whole body of the thesis does not feel like one connected theme/idea, but rather like two or three separate ideas, presented in different chapters.

5 Strengths

There seem to me (on my reading) no major scientific errors in the thesis. The explanations are clear and at the right level. The references / bibliography is wonderful and comprehensive. The references therein are linked in correctly in the narrative: I feel if I had to follow in more detail any of the ideas in the thesis, I would not be let down by the links to the bibliography. As pointed out by the author, most of the work has been published in international peer-reviewed journals (List of publications on page 9), and so, apart from my analysis of the work, it has also been refereed by other experts/peers for these journals. The author makes original contributions, most strongly in quantum annealing for music theory, but also in the homotopy and error mitigation QAOA. While it is arguable (and probably subjective), the cognitive significance is higher for the music-related-work, while the practical relevance higher for the homotopy and error mitigation QAOA.

6 Corrections

Separate from this report, I send a second document with detailed corrections (100+). These can be broadly placed in two categories:

1. Small grammar corrections (about half of all the errors). In nearly all cases, the corrected grammar does not change the meaning in a significant way (in other words, even if this was not corrected, a reader would most likely understand what is being said). Nonetheless, it is good to have correct English!
2. Other corrections - in scientific aspects of the English, or acronyms, or of course in mathematical notation.

7 Conclusion and Recommendation

I conclude that this is a solid and novel piece of research, deserving of the award of a doctoral degree in the field of engineering and technology, in the discipline of information and communication technology. I pass the dissertation, but do not recommend it should be "distinguished". I recommend nonetheless that all the corrections (listed separately) be made by the author.