



# Performance of a Buffer Between Electronic and All-Optical Networks, Diffusion Approximation Model

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**Abstract.** We present a model of the edge router between electronic and all optical networks. Arriving electronic packets of variable sizes are stored at a buffer the volume of which is equal to the fixed size of optical packet. When the buffer is filled, its content becomes an optical buffer and dispatched to optical network. To avoid excessive delays, the optical packet is sent also after a specified deadline. The model is based on diffusion approximation and validated by discrete event simulation. Its goal is to determine the probability distribution of the effective optical packet sizes and distribution of their interdeparture times as a function of the interarrival time distribution, distribution of the electronic packet sizes, the value of deadline and the size of buffer. We use real traffic data from the CAIDA (Center for Applied Internet Data Analysis) repositories.

## 1 Introduction

In all-optical networks which are still a technology under development, switching is performed by optical nodes and the optical signal is not converted to electronic form along the whole way through the network. This may improve substantially the network performance. However, it is not easy to organize. Electronic nodes are able to queue and prioritize packets, to decide the further routing, while the optical ones may only, if needed, delay the signal in special fibre loops. That is why the ingress router should do as much work as possible: incoming electronic packets are here sorted by destination and by class of service (e.g. CoS1, CoS2, Best Effort). We consider the performance of a single buffer where packets of the same destination and class are stored.

The high demand for more bandwidth and high speed networks have motivated a lot of research in the design, optimization and performance evaluation of IP over all-optical networks [6]. There is a great need to improve the efficiency of the packetization process since the delay in IP over all optical network is the

sum of the delays in the packetization process, the delays in the buffer where the packets are scheduled into the all optical network, the propagation delays in the optical transmission links and the delays in the optical nodes.

Here, we use diffusion approximation which is an analytical tool developed to evaluate the performance of queueing systems with general arrival and general service time distributions, [12, 13]. The diffusion process represents here the process of assembling and framing electronic IP packets into optical packets. Our study is based on the packetization model proposed in [7] which we formerly studied in [6] using Markovian model solved numerically with a Probabilistic Model Checker (Prism).

The diffusion models are validated by a discrete event simulation model developed using the JAVA programming language. We use a synthetic traffic in form of Poisson process and real traffic traces from the CAIDA repositories. The performance measures we consider are the distribution of optical packet inter-departure times (delays introduced by packetization and regularity of traffic at optical side) and the distribution of the size of content in optical packets which are of constant size but may be only partially filled (throughput).

## 2 Packetization in IP over All-Optical Networks

Packetization in IP over all optical networks is the process of grooming smaller electronic IP packets into larger optical packets. It is carried out in the edge node where the incoming IP packets are classified based on their QoS classes, destination and other relevant parameters, queued in a buffer until the buffer is full or until the filling deadline expires. This has given rise to two types of packetization algorithms adopted in IP over all optical networks. These are the maximum time (Max Time) and the maximum time maximum size (Max Time Max Size) algorithms described in detail in [8]. In the Max Time algorithm, after a predefined deadline  $T$ , the content of the buffer is emptied while in the Max Time Max Size, the buffer is emptied after the predefined deadline has expired or after the maximum size of the buffer  $N$  is reached, even if the deadline has not yet expired.

The authors in [4, 6, 7] adopted the Max Time Max Size algorithm but with the modification that if the mean size of the arriving IP packet say  $m$ , is larger than the free available space in the buffer, the buffer is considered to be full, its content is dispatched as an optical packet and the rejected IP packet is rescheduled in the next filling cycle. The authors in [9] proposed two adaptive packetization (assembly) algorithms which are packet-based dynamic-threshold algorithm for burst assembly (dyn-threshold-packet) and the byte-based dynamic threshold algorithm for burst assembly. They used synthetic traffic (Poisson traffic and Markov Modulated Poisson Process traffic) and the real traffic traces from a traffic data repository maintained by the measurement and analysis on the WIDE Internet (MAWI) working group of the WIDE Project to demonstrate that the dynamic threshold-based assembly algorithms perform substantially better than the usual timer-based schemes. The authors in [14] are proposing a

packetization scheme with adaptive expiration times, determined in response to local and/or global queue sizes.

In this paper, we considered the first two criteria above and then solve diffusion equations without considering the influence of the deadline (MTMS-S) and in another case without considering the influence of the buffer size (MTMS-T), that is considering the influence of the deadline. However, we still considered the packetization mechanism that we used in [6], alongside the first two criteria.

### 3 Simulation Model

A discrete event simulation model written in Java was prepared. In simulation we used traffic traces from CAIDA (Center for Applied Internet Data Analysis) [1] repositories. CAIDA routinely collects traces on several backbone links, in the examples that follow we used measurements of the size of IP v4 packets and their interarrival times from Equinix Chicago link collected during one hour on 18 February 2016, having 22 644 654 packets belonging to 1 174 515 IPv4 flows, [2].

In the model we compare the performance of the buffer in presence of the resulting intensity of CAIDA traffic flow (in bytes per time unit) with Poisson traffic of the same intensity. The dispersion of the real traffic intensity is clearly larger and limited only by the maximum throughput of the link. The time unit is 0.01 sec, in this scale the burstiness of traffic is seen the best. The same time unit was then applied in diffusion model.

The assumed size of the buffer was  $N = 10000$  bytes, corresponding to the size of about 15 (14.32) average packets or 6.66 maximum packets size. The simulation results concerning the distribution of interdeparture times of optical packets and their filling without and with deadline  $T = 0.02$  s are in next sections compared with diffusion approximation results.

### 4 Diffusion Approximation Model Without Deadline

In this section we develop diffusion approximation model for the interdeparture times of optical packets with no deadline. Since the incoming IP packets are queued up in the buffer until the buffer is full we can treat this system as a queueing system with mean interarrival rate  $\lambda$  and mean service rate  $\mu = 0$ .

The principle of the diffusion approximation is to replace the number of customers in a queueing system by the value of diffusion process  $X(t)$ , [12, 13]. The solution of the diffusion equation

$$\frac{\partial f(x, t; x_0)}{\partial t} = \frac{\alpha}{2} \frac{\partial^2 f(x, t; x_0)}{\partial x^2} - \beta \frac{\partial f(x, t; x_0)}{\partial x} \quad (1)$$

where  $\beta dt$  and  $\alpha dt$  represent the mean and variance of the changes of the diffusion process at  $dt$ , defines the conditional probability density function (pdf) of the diffusion process  $f(x, t; x_0) = P[x \leq X(t) < x + dx \mid X(0) = x_0]$  of  $X(t)$ .

The density of the unrestricted diffusion process starting from  $x_0$  is, e.g. [3]

$$\phi(x, t; x_0) = \frac{1}{\sqrt{2\pi\alpha t}} \exp\left[-\frac{(x - x_0 - \beta t)^2}{2\alpha t}\right] \quad (2)$$

and in case of starting the process at  $x_0 = 0$  and having the absorbing barrier at  $x = N$  (i.e. the process is ended when it comes to the barrier,  $f(N, t) = 0$  for  $t > 0$ ), is

$$\phi(x, t) = \frac{1}{\sqrt{2\pi\alpha t}} \left\{ \exp\left[-\frac{(x - \beta t)^2}{2\alpha t}\right] - \exp\left[\frac{2\beta N}{\alpha} - \frac{(x - 2N - \beta t)^2}{2\alpha t}\right] \right\}. \quad (3)$$

The first passage time from  $x = 0$  to the barrier has density  $\gamma_{0,N}(t)$

$$\gamma_{0,N}(t) = -\frac{d}{dt} \int_{-\infty}^N \phi(x, t) dx = \frac{N}{\sqrt{2\pi\alpha t^3}} \exp\left[-\frac{(N - \beta t)^2}{2\alpha t}\right]. \quad (4)$$

This first passage time approximates the time of filling the buffer and also the interdeparture time between optical packets. The filling of the buffer starts at  $x = 0$  when  $t = 0$ , and ends at the barrier at  $x = N$  where  $N$  corresponds to the size of the buffer. The value  $x$  of the diffusion process at time  $t$  represents the current filling of the buffer.

The number of bytes arrived at a unit of time is a product of two independent random variables:  $X$  – the number of packets and  $Y$  – the size of packets. The mean of a product variable  $XY$  is  $E(XY) = E(X)E(Y)$  and the variance is

$$\begin{aligned} \text{Var}(XY) &= E(X^2Y^2) - (E(XY))^2 = \\ &= \text{Var}(X)\text{Var}(Y) + \text{Var}(X)(E(Y))^2 + \text{Var}(Y)(E(X))^2, \end{aligned}$$

the mean number of arrived at a time unit packets is  $E(X) = \lambda$  and the variance is  $\text{Var}(X) = \lambda^3\sigma_A^3$ , therefore the mean of arrived at time unit bytes is  $\beta = \lambda m$  and the variance of number of arrived at a time unit bytes that define  $\alpha$  is

$$\alpha = \lambda^3\sigma_A^2\sigma_m^2 + \lambda^3\sigma_A^2m^2 + \sigma_m^2\lambda^2$$

We may refine this model having in mind that the interdeparture time includes time the buffer stays empty, that means completion time of the interarrival time (below, for simplicity we take the interarrival time distribution  $f_A(x)$  of packets after which the diffusion is started not at zero but at a point corresponding to the size of the first arriving packet, i.e. at  $x_0 = \xi$  given by the distribution  $f_H(\xi)$  of the packet size:

$$\gamma(t) = f_A(t) * \int_0^H \gamma_{\xi,N}(t) f_H(\xi) d\xi$$

where  $*$  is the convolution operation.

## 5 Diffusion Approximation Model with Deadline

If we do not consider the deadline, the optical packets leaving the buffer are always full. In case of deadline  $T$ , the expression (4) of the first passage time evaluates the density of probability  $\gamma_{0,x}(x)$

$$\gamma_{0,x}(x) = -\frac{x}{\sqrt{2\pi\alpha T^3}} \exp\left[-\frac{(x - \beta T)^2}{2\alpha T}\right] \quad (5)$$

that the filling of the buffer ends at position  $x$  (provided that the process was not ended on its way to  $x$ ) because of the deadline  $T$ . Dispatched this way the optical packet contains  $x$  bytes, the remaining  $N - x$  of its volume being empty: The probability density that the diffusion process will end exactly when the deadline  $T$  is reached is given by:

$$\gamma_{0,N}(T) = \int_T^\infty \gamma_{0,N}(t) dt = N \left[ 2 - \operatorname{erfc}\left(\frac{N - T\beta}{\sqrt{2T\alpha}}\right) + e^{\frac{2N\beta}{\alpha}} \operatorname{erfc}\left(\frac{N - T\beta}{\sqrt{2T\alpha}}\right) \right] \quad (6)$$

where

$$\operatorname{erfc}(t) = 1 - \operatorname{erf}(t), \quad \operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-\xi^2} d\xi.$$

Figure 1 compares the interdeparture times obtained by simulation and diffusion approximation in case of deadline and Poisson input. The influence of deadline is hardly seen as the passage time in case of Poisson traffic are usually shorter than  $T$ . In case of more dispersed real traffic the passage time may be larger and probability that the filling ends with deadline, expressed by Eq. 6 is distinct. The solutions for the model without deadline are quite similar to those in the figures except spikes resulting from deadline.

When the deadline is reached, the buffer should be cleared and the diffusion equation should be provided with jumps performed from the point  $x$  to  $x = 0$  with the density  $\gamma_{0,x}(x)$ . Some similar models of diffusion with jumps back were discussed in [10]. Here, the diffusion equation

$$\frac{\partial f(x, t, x_0)}{\partial t} = \frac{\alpha}{2} \frac{\partial^2 f(x, t, x_0)}{\partial x^2} + \beta \frac{\partial f(x, t, x_0)}{\partial x} - \gamma f(x, t, x_0) \quad (7)$$

in steady state becomes a second order homogeneous linear differential equation with solution

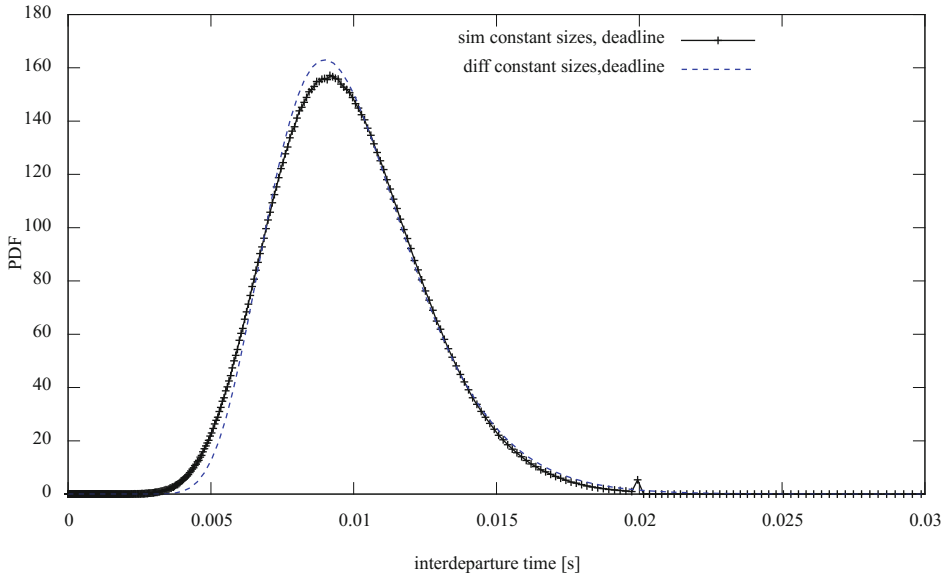
$$f(x) = C_1 e^{z_1 x} + C_2 e^{z_2 x}$$

where  $z_1, z_2$

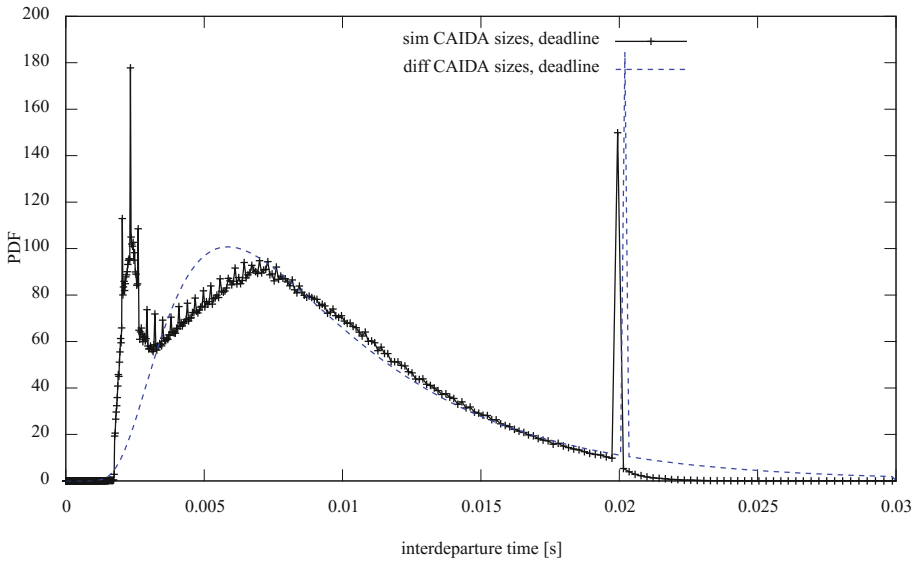
$$z_1 = \frac{\beta}{\alpha} + W, \quad z_2 = \frac{\beta}{\alpha} - W, \quad W = \sqrt{\frac{\beta^2}{\alpha^2} + \frac{2\gamma}{\alpha}}$$

are the roots of the characteristic polynomial

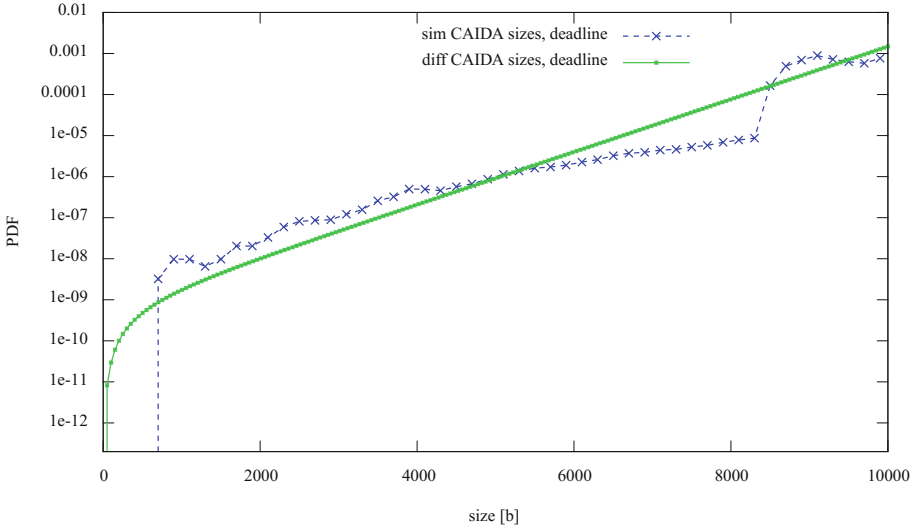
$$\frac{\alpha}{2} z^2 - \beta z - \gamma = 0$$



**Fig. 1.** Interdeparture times, simulation and diffusion approximation with deadline (Poisson input).



**Fig. 2.** Interdeparture times, simulation and diffusion approximation with deadline (CAIDA input).



**Fig. 3.** Distribution of packet sizes in case of deadline, diffusion approximation and simulation, (CAIDA input).

Using the border condition  $\lim_{x \rightarrow 0} f(x) = 0$  ( $C_1 = -C_2$ ) and the normalization  $\int_0^N f(x) dx = 1$ , we have

$$f(x) = C(e^{z_2 x} - e^{z_1 x}), \quad C = z_2 e^{-z_2 N} - z_1 e^{-z_1 N}. \tag{8}$$

We use this solution to approximate the probability that the the buffer is filled up to the value  $x$  when the deadline comes with density given by Eq. 5. Distribution of effective packet sizes, given by diffusion approximation and simulation real (CAIDA) traffic is displayed in Fig. 3.

## 6 Conclusions

In a comparatively easy way diffusion approximation gives reasonable model of interdeparture time distribution and of useful packet size after framing electronic packets to optical packet at the electronic-optical edge node. The model may be refined to capture the influence of the insufficient space when the buffer is almost full and the incoming packet starts a new filling cycle.

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